

## Four generation $CP$ violation in $B \rightarrow \phi K^0, \pi^0 K^0, \eta' K^0$ and hadronic uncertainties

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ABSTRACT: The fourth generation can give the correct trend of  $\mathcal{S}_{\phi K^0}, \mathcal{S}_{\pi^0 K^0} < \sin 2\phi_1$ , as indicated by data, and the effect, being largely leading order, is robust against hadronic uncertainties. The effect on  $\mathcal{S}_{\eta' K^0}$ , however, is diluted away by hadronic effects, and  $\mathcal{S}_{\eta' K^0} \simeq \sin 2\phi_1$  is expected. The near maximal  $\arg V_{t's}^* V_{t'b} \lesssim 90^\circ$  that is needed could resolve the unequal direct  $CP$  violation seen in  $B \rightarrow K^+ \pi^-$  and  $K^+ \pi^0$  modes, and is consistent with  $b \rightarrow s \ell^+ \ell^-$  and  $B_s$  mixing constraints.

KEYWORDS: B-Physics, Beyond Standard Model, CP violation, Rare Decays.

At present there are two hints for possible New Physics (NP) from  $CP$  violation (CPV) studies in the  $B$  system, both in charmless  $b \rightarrow s$  transitions.

Time dependent CPV (TCPV) in  $B$  decays to  $CP$  eigenstate  $f$  is measured by

$$\frac{\Gamma(\overline{B}^0(t) \rightarrow f) - \Gamma(B^0(t) \rightarrow f)}{\Gamma(\overline{B}^0(t) \rightarrow f) + \Gamma(B^0(t) \rightarrow f)} = S_f \sin(\Delta m_B t) - C_f \cos(\Delta m_B t). \quad (1)$$

We concern ourselves with  $S_f$  only, since  $-C_f = A_f$  are all consistent with zero so far. The current world average of  $S_f$  in  $b \rightarrow c\bar{c}s$  decays gives  $\sin 2\phi_1 = 0.69 \pm 0.03$  [1], which dominantly comes from  $B^0 \rightarrow J/\psi K_S$ . TCPV measurements in loop dominated  $b \rightarrow s\bar{q}q$  processes such as  $B^0 \rightarrow \eta' K^0$ ,  $\phi K^0$  and  $\pi^0 K^0$ , on the other hand, have persistently given values below  $\sin 2\phi_1$ . The current values [1] are  $\mathcal{S}_{\eta' K^0} = 0.50 \pm 0.09$ ,<sup>1</sup>  $\mathcal{S}_{\phi K^0} = 0.47 \pm 0.19$  and  $\mathcal{S}_{\pi^0 K^0} = 0.31 \pm 0.26$ .

It was suggested some time ago [2] that, for  $b \rightarrow s\bar{q}q$  final states, a significant  $\Delta\mathcal{S}_f = \mathcal{S}_f - \sin 2\phi_1$  would indicate NP. The study of theoretical uncertainties for  $\Delta\mathcal{S}_f$  has therefore been a great focus during the past year. Consensus has emerged that  $\Delta\mathcal{S}_f$  in these modes tend to be small and *positive* [3–5] within the Standard Model (SM), which is opposite the trend seen by experiment. It is therefore imperative to establish  $\Delta\mathcal{S}_f \neq 0$  experimentally beyond any doubt in a few modes, which would require considerably more data than present.

A simpler measurement than  $\mathcal{S}_f$  is direct CPV (DCPV) asymmetries in flavor-specific final states, which does not require time dependent measurement. DCPV was recently observed [6] in  $B^0 \rightarrow K^+\pi^-$  decay, i.e.  $\mathcal{A}_{K^+\pi^-} = -0.115 \pm 0.018$ . Having similar dominating penguin and tree contributions, one would naively expect that  $\mathcal{A}_{K^+\pi^-} = \mathcal{A}_{K^+\pi^0}$ . However, no indication of DCPV was seen in charged  $B^+ \rightarrow K^+\pi^0$ , i.e.  $\mathcal{A}_{K^+\pi^0} = 0.04 \pm 0.04$ . The difference with  $\mathcal{A}_{K^+\pi^-}$  could be due to an enhanced color-suppressed amplitude  $C$  [7], or electroweak penguin  $P_{EW}$  effects [8–11]. The former requires  $C$  to effectively cancel the SM phase in color-allowed tree amplitude  $T$ , without recourse to NP. For the latter, NP CPV phases would be needed in  $P_{EW}$ .

It would be intriguing if the two hints of NP, one in  $\Delta\mathcal{S}_f \neq 0$ , the other in  $\mathcal{A}_{K^+\pi^0} - \mathcal{A}_{K^+\pi^-} \neq 0$ , could be manifestations of the same NP source. Since  $\pi^0$  and  $\phi$  (but not  $\pi^-$ ) can materialize from a virtual  $Z$ , the  $B \rightarrow \pi^0 K^0$ ,  $\phi K^0$  modes are sensitive to  $Z$  penguins. The effect of a NP phase in  $P_{EW}$  on  $\Delta\mathcal{S}_f$ , among several NP scenarios, was studied in ref. [5]. In another work, some of us have shown [11] that the 4th generation could provide a solution to the  $\mathcal{A}_{K^+\pi^0} - \mathcal{A}_{K^+\pi^-} \neq 0$  problem through the electroweak penguin. The 4th generation can make specific impact on  $P_{EW}$  because the  $t'$  quark, like the SM top, enjoys nondecoupling in  $P_{EW}$  [12], but largely decouples from photonic and gluonic penguins. Furthermore, it can provide a new CPV phase [13] through  $V_{t's}^* V_{t'b} \equiv r_{sb} e^{i\phi_{sb}}$ .

In this work we show that the fourth generation can, for the right choice of  $\phi_{sb}$ , give the correct trend for  $\Delta\mathcal{S}_f$  in  $B^0 \rightarrow \pi^0 K^0$  and  $\phi K^0$ , and is robust against hadronic uncertainties.

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<sup>1</sup> The BaBar value of  $\mathcal{S}_{\eta' K^0} = 0.36 \pm 0.13 \pm 0.03$ , and the Belle value of  $\mathcal{S}_{\eta' K^0} = 0.62 \pm 0.12 \pm 0.04$  are in some variance. One could inflate the error of the mean by a factor of  $\sqrt{2}$ . We refrain from doing so and use the value quoted by HFAG, for sake of consistency with other modes. Note that the Belle value is in good agreement with  $\sin 2\phi_1$ .

$i$	$C_i^t(m_b)$	$\Delta C_i(m_b)$
1	1.086	-
2	-0.191	-
3	1.353	0.316
4	-3.534	-0.155
5	0.982	0.024
6	-4.257	-0.134
7	-0.007	0.215
8	0.025	0.080
9	-0.990	-0.972
10	0.210	0.221

**Table 1:** Numerical values of the Wilson coefficients at  $m_b$  scale.  $C_i^t$  ( $i = 1-10$ ) stands for the  $t$  effects, while  $\Delta C_i$  is the  $t'$  effects for  $m_{t'} = 300$  GeV. The values for  $i = 3-10$  are in unit of  $10^{-2}$ .

In contrast, we find  $\Delta \mathcal{S}_{\eta' K^0}$  is largely diluted by hadronic effects that are needed to account for the large rate.

For relevant 4th generation parameters, we take [11]

$$m_{t'} = 300 \text{ GeV}, \quad r_{sb} \simeq 0.025, \tag{2}$$

and vary  $\phi_{sb}$  phase. Eq. (2) is consistent with  $b \rightarrow s\ell^+\ell^-$  and  $B_s$  mixing constraints [11]. Larger  $m_{t'}$  or  $r_{sb}$  could lead to larger effects on  $\Delta S_f$ , but could run into trouble with the other  $b \rightarrow s$  constraints. Naively the NP effect scales like  $r_{sb} \times m_{t'}^2/M_W^2$ . So a positive change in  $r_{sb}$  needs always to be accompanied by a negative change in  $m_{t'}$  and viceversa. To study (factorization) model dependence, we compare results in naive factorization (NF) [14], QCD factorization (QCDF) [15, 16] and PQCD [17]. We further use QCDF to illustrate hadronic uncertainties. We choose to use QCDF and PQCD circa 2003 because, in part stimulated by the  $\Delta \mathcal{S}$  and  $\mathcal{A}_{K^+\pi^0}$  problems, these factorization models are still being refined.

We adopt QCDF as our reference framework. Defining  $\lambda_i \equiv V_{is}^* V_{ib}$ , one has  $\lambda_u + \lambda_c + \lambda_t + \lambda_{t'} = 0$  with existence of  $t'$ . To good approximation,  $\lambda_u$  is negligible compared with  $\lambda_c \simeq 0.04$ , where we have taken the convention to keep  $V_{cb}$  real, and placing the 3 CPV phases in  $V_{ub}$ ,  $V_{t's}$  and  $V_{t'd}$  [18, 19]. This makes clear correspondence to the standard phase convention for 3 generation case. The unitarity condition  $\lambda_t \simeq -\lambda_c - \lambda_{t'}$  allows one to absorb the  $t$  effect into the  $\lambda_c$  dependent part (SM term), and the NP  $\lambda_{t'}$  dependent part that respects GIM [13, 11].

The  $\overline{B}^0 \rightarrow \pi^0 \overline{K}^0$  amplitude is

$$\begin{aligned} \mathcal{M}_{\pi^0 \overline{K}^0} \propto & f_\pi F_{BK} \left( \lambda_u a_2^u + \frac{3}{2} \lambda_c \alpha_{3,\text{EW}}^p - \frac{3}{2} \lambda_{t'} \Delta \alpha_{3,\text{EW}}^p \right) \\ & - f_K F_{B\pi} \left[ \lambda_c \left( \alpha_4^p - \frac{1}{2} \alpha_{4,\text{EW}}^p + \beta_3^p \right) + \frac{1}{2} \lambda_{t'} \Delta \alpha_{4,\text{EW}}^p \right], \end{aligned} \tag{3}$$

where  $\alpha_{i(\text{EW})}^p$  and  $\beta_3^p$  are defined in ref. [15] and evaluated for the  $\pi^0 \overline{K}^0$  final state, and  $\Delta \alpha_{i,\text{EW}}^p$  is the effective ( $t$  subtracted)  $t'$  contribution. Table I enumerates the SM values

for the Wilson coefficient  $C_i^{t'}$  and the  $t'$  correction  $\Delta C_i (\equiv C_i^{t'} - C_i^t)$  at  $m_b$  scale. In our calculation, we follow ref. [20] and take  $m_b = 4.2$  GeV,  $m_t = 167$  GeV, and  $m_{t'} = 300$  GeV. The  $m_{t'}$  dependence of  $\Delta C_i$  can be found in ref. [11].

For  $\overline{B}^0 \rightarrow \phi \overline{K}^0$ , there is no tree term, and one has

$$\begin{aligned} \mathcal{M}_{\phi \overline{K}^0} \propto \lambda_c \left[ \alpha_3^p + \alpha_4^p + \beta_3^p - \frac{1}{2} \left( \alpha_{3,\text{EW}}^p + \alpha_{4,\text{EW}}^p + \beta_{3,\text{EW}}^p \right) \right] \\ + \frac{\lambda_{t'}}{2} (\Delta \alpha_{3,\text{EW}}^p + \Delta \alpha_{4,\text{EW}}^p + \Delta \beta_{3,\text{EW}}^p), \end{aligned} \quad (4)$$

where  $\alpha_{i,\text{EW}}^p$  and  $\beta_3^p$  are evaluated for the  $\phi \overline{K}^0$  final state. We have dropped the common  $f_\phi F_{BK}$  factor compared to eq. (4), and we show only the more important terms. The numerics was done with full details according to ref. [16]. The formula for  $\overline{B}^0 \rightarrow \eta' \overline{K}^0$  can be analogously written, but is more elaborate which we do not reproduce here. We stress that the same expressions apply to the amplitudes in NF framework as well, with the various coefficients taken at LO instead of NLO.

In this work we estimate and quantify the impact of hadronic uncertainties in QCDF. Among the hadronic parameters that enter the decay amplitudes, three stand out as having the largest impact due to uncertainties [15]: the divergent part of the hard spectator scattering integral  $X_H$ , the divergent part of the weak annihilation integral  $X_A$ , both entering power suppressed terms by  $\Lambda_{\text{QCD}}/m_b$ , and the first inverse moment of the  $B$  meson distribution amplitude  $\lambda_B$ . The first two are estimated to be complex numbers of order  $\ln(m_B/\Lambda_h)$  with  $\Lambda_h = 500$  MeV, and can therefore be parameterized by<sup>2</sup>

$$X_{H,A} = \left( 1 + \rho_{H,A} e^{i\phi_{H,A}} \right) \ln \frac{m_b}{\Lambda_h}. \quad (5)$$

Our estimate of the hadronic uncertainties is based on the variation of these parameters over a wide range as indicated in ref. [15]. For reference, we also take as baseline a “standard” scenario, in which we fix  $\rho_H = 0$ ,  $\rho_A = 1$ ,  $\phi_A = -45^\circ$  and  $\lambda_B = 350$  MeV. This scenario corresponds to the “S3” scenario of ref. [16], although small numerical differences in input parameters may lead to a slight difference in final results [21].

For  $\overline{B}^0 \rightarrow \pi^0 \overline{K}^0$  in PQCD factorization, we adopt the LO result used in ref. [11]

$$\mathcal{M}_{\pi^0 \overline{K}^0} \propto \lambda_u f_\pi F_{ek} + \lambda_c (-f_K F_e^P - f_B F_a^P + f_\pi F_{ek}^P) - \lambda_{t'} f_\pi \Delta F_{ek}^P, \quad (6)$$

where  $F_e^P$ ,  $F_a^P$ ,  $F_{ek}$  and  $F_{ek}^P$  are the strong penguin, strong penguin annihilation, color suppressed tree and (color allowed) electroweak penguin contributions, respectively. These factorizable contributions can be computed by following ref. [17], and are tabulated in ref. [11].

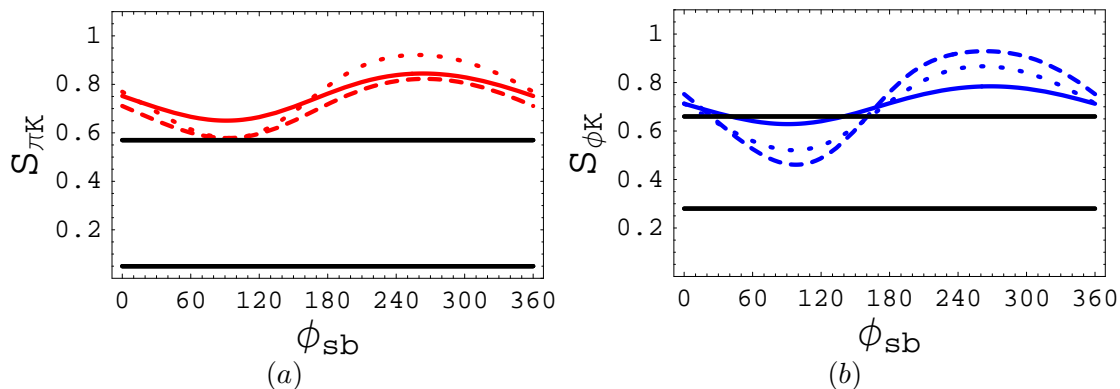
For  $\overline{B}^0 \rightarrow \phi \overline{K}^0$ , we have

$$\mathcal{M}_{\phi \overline{K}^0} \propto \lambda_c (f_\phi F_e^P + f_B F_a^P) - \lambda_{t'} f_\phi \Delta F_e^P, \quad (7)$$

where the  $F_i^P$ s are evaluated for  $\phi \overline{K}^0$  [22] and not the same as in eq. (6). We have performed

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<sup>2</sup>In principle both  $X_H$  and  $X_A$  are mode dependent. However, since we vary them over the whole possible range in any case, this point does not affect our analysis.



**Figure 1:** (a)  $S_{\pi^0 K^0}$ , (b)  $S_{\phi K^0}$  vs  $\phi_{sb}$  for QCDF at NLO with “S3” parameters (solid), PQCD at LO (dashed), and NF (dots), which is QCDF at LO. The horizontal solid band is the current experimental range.

only an approximate computation in this case. We assume that the scale  $t$ , where the Wilson coefficients are evaluated, has a mild dependence on the momentum fraction  $x$  and the impact parameter  $b$  which is conjugate to the parton transverse momentum. The amplitude  $F_e^P$ , which is obtained by integrating over the variables  $x$  and  $b$ , becomes then proportional to  $a_e(t)$  with

$$a_e(t) = C_3 + \frac{C_4}{3} + C_4 + \frac{C_3}{3} + C_5 + \frac{C_6}{3} - \frac{1}{2} \left( C_7 + \frac{C_8}{3} + C_9 + \frac{C_{10}}{3} + C_{10} + \frac{C_9}{3} \right). \quad (8)$$

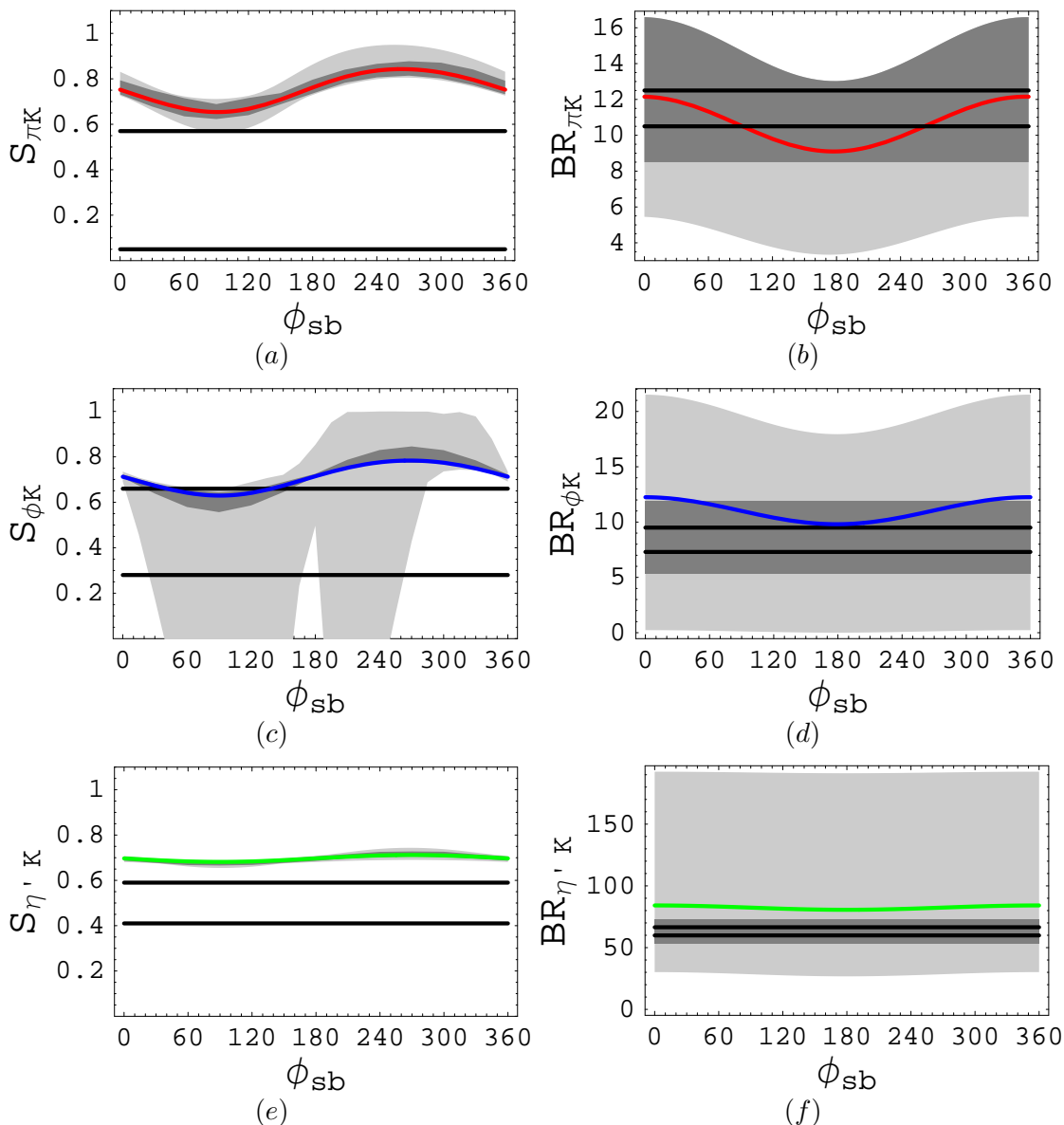
By knowing now the numerical value of  $F_e^P$  [17] and the Wilson coefficients in the SM calculated at  $t = m_b$ , one can then calculate  $\Delta F_e^P$  with

$$\Delta F_e^P = F_e^P \left( \frac{a_e^{NP}}{a_e^{SM}} - 1 \right). \quad (9)$$

The same procedure is not possible for  $F_a^P$  and we keep only the SM contribution by assuming  $\Delta F_a^P = 0$ .<sup>3</sup> For  $\eta' K$  mode, not much work has been done in PQCD.

To study the model dependence in different factorization approaches, we plot  $S_{\pi^0 K^0}$  and  $S_{\phi K^0}$  vs  $\phi_{sb}$  in figure 1 for QCDF at NLO (with “S3” parameters), PQCD at LO, and NF. The latter is far from realistic (for rates) and is just for comparison. We see that in all three models,  $S_{\pi^0 K^0}$  and  $S_{\phi K^0}$  dip below  $\sin 2\phi_1$  for  $\sin \phi_{sb} \gtrsim 0$ , especially around  $\phi_{sb} \sim 90^\circ$ . Indeed, for a given size of NP contribution, a choice of a maximal weak phase of  $90^\circ$  (or  $270^\circ$ ) tends to maximize the NP effect on CPV while minimizing the NP effect on BR. It is interesting to note that this is precisely what is needed for the 4th generation to help resolve [11] the  $\mathcal{A}_{K+\pi^0} - \mathcal{A}_{K+\pi^-} \neq 0$  problem. Independently,  $\phi_{sb} \sim 90^\circ$  is also the parameter space where  $b \rightarrow s\ell^+\ell^-$  and  $\Delta m_{B_s}$  constraints are best evaded [11, 13].

<sup>3</sup>Since one expects very strong GIM cancellation between  $t'$  and  $t$  for strong penguin, the approximation should be a very good one.



**Figure 2:** (a)  $S_{\pi^0 K^0}$ , (b)  $\mathcal{B}(\pi^0 K^0)$ , (c)  $S_{\phi K^0}$ , (d)  $\mathcal{B}(\phi K^0)$ , (e)  $S_{\eta' K^0}$ , (f)  $\mathcal{B}(\eta' K^0)$  vs  $\phi_{sb}$ , in QCDF at NLO. The curves are for the “S3” hadronic parameter settings. The light gray regions correspond to varying the hadronic parameters  $X_A$ ,  $X_H$  and  $\lambda_B$  over the range indicated in eq. (10). The dark gray regions are obtained by varying the hadronic parameters over the same range as above, but keeping only the values for which the branching ratios are within  $3\sigma$  of the experimental central values.

For  $\phi_{sb} \sim 270^\circ$ , although the  $b \rightarrow s\ell^+\ell^-$  and  $\Delta m_{B_s}$  constraints can still be tamed, both  $\mathcal{S}_{\pi^0 K^0, \phi K^0}$  and  $\mathcal{A}_{K^+\pi^-}$  would be in disagreement with experiment.

Hadronic parameters such as strong phases easily affect branching ratios and DCPV asymmetries. Strong phases are definitely present in  $B \rightarrow K\pi$  decay as evidenced by the sizable  $\mathcal{A}_{K^+\pi^-} = -0.115 \pm 0.018$ . The  $S_f$  parameter, however, measures the weak phase

of the decay amplitude, and is less affected by hadronic parameters [5]. As mentioned, we illustrate this point, by varying the hadronic parameters of QCDF at NLO around the “S3” scenario settings. In particular, we vary [15, 16]

$$\begin{aligned} \rho_{A,H} &\in (0, 1), \quad \phi_{A,H} \in [0, 2\pi], \\ \lambda_B &\in [200, 500]\text{MeV}, \end{aligned} \tag{10}$$

for the  $X_A$ ,  $X_H$  and  $\lambda_B$  parameters.

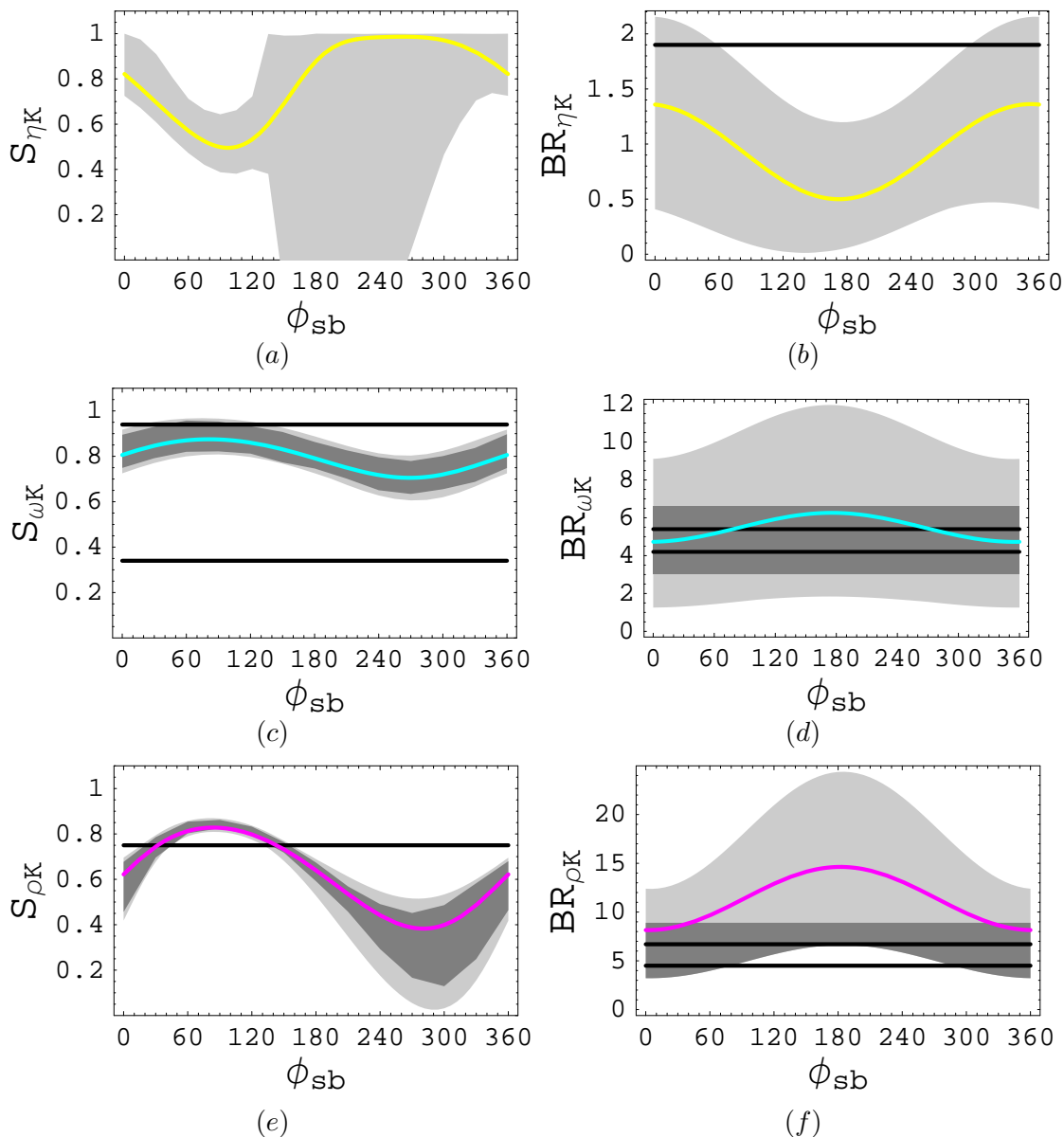
We plot  $S_{\pi^0 K^0}$ ,  $S_{\phi K^0}$  and  $S_{\eta' K^0}$  vs  $\phi_{sb}$  in the left side of figure 2. The light shaded regions correspond to varying the parameters over the whole range indicated in eq. (10). The dark shaded regions correspond to varying the hadronic parameters over the same range, but keeping, for each mode, only the values that produce a branching ratio (right side of figure 2) within  $3\sigma$  of the experimental central value. One sees that, indeed, the branching ratios are strongly affected by the hadronic parameters, and most of the hadronic parameter space cannot survive the bulk of rate and DCPV data when considered together. In contrast, the range of variation for  $S_f$  is much more subdued. This is encouraging: the NP effect in  $S_f$  for the  $\pi^0 K^0$ ,  $\phi K^0$  and  $\eta' K^0$  modes is robust.

We note that the effect of hadronic parameters, when varied over the whole range, is rather strong for  $S_{\phi K^0}$ . However, when the experimental constrains on the  $B \rightarrow \phi K^0$  branching ratio are taken into account, the hadronic uncertainty in  $S_{\phi K^0}$  is highly diminished. Note, also, that  $S_{\eta' K^0}$  gets strongly diluted away. The reason behind this is the rather large rate of  $B \rightarrow \eta' K$  decay, which seemingly draws from  $CP$  conserving (“hadronic”) effects, since there is little evidence for CPV i.e.  $\mathcal{A}_{\eta' K} \simeq 0$  [1]. Furthermore, the  $Z$  penguin contribution has relatively small strength.

We believe the dilution of  $S_{\eta' K^0}$  is a generic effect, that is, it is very hard for NP CPV effects to shine through the large hadronic effects, and  $S_{\eta' K^0} \simeq \sin 2\phi_1$  should be expected. In this respect, the Belle result of  $\mathcal{S}_{\eta' K^0} = 0.62 \pm 0.12 \pm 0.04$ , which is fully consistent with  $\sin 2\phi_1 = 0.69 \pm 0.03$ , is easier to explain in most NP models.<sup>1</sup> If the BaBar result holds out eventually, it would need some conspiracy between NP and hadronic effects to realize theoretically.

For completeness we give in figure 3 the values of  $S$  and  $\mathcal{B}$  for the modes  $\eta K^0$ ,  $\omega K^0$  and  $\rho K^0$  and in figure 4 the DCPV parameter  $C$  for all the modes consider in this paper.

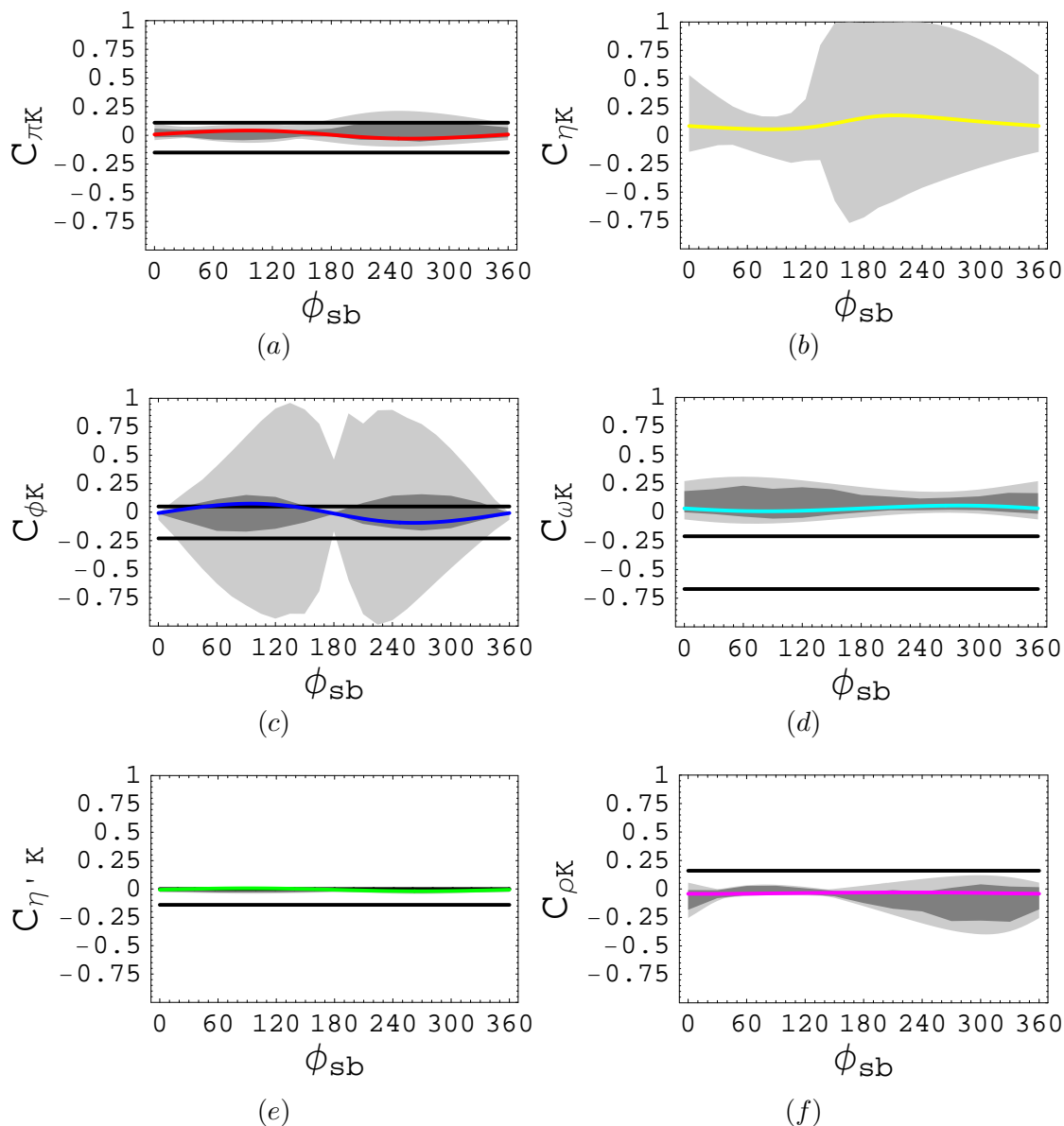
We offer some remarks before closing. We have studied the ratio of branching ratios  $R_c$ ,  $R_n$  and  $R$ , which are for  $B^+$ ,  $B^0$ , and the lifetime corrected  $K^+\pi^-$  over  $K^0\pi^+$  ratio, respectively. Indeed, these rate ratios are attractive in that they suffer considerably less hadronic uncertainties. We plot  $R_c$  and  $R_n$  vs  $\phi_{sb}$  in figure 5 for QCDF at NLO and varying hadronic parameters over the full range of eq. (10). The contrast with the branching ratio plot in figure 2 is striking. Interestingly, for  $|\phi_{sb}| \lesssim 80^\circ$ , the hadronic uncertainties are even less significant (a bit more for  $R$ ), and the results with 4th generation are basically consistent with experiment. But for  $\phi_{sb} \sim \pi$ , besides much larger hadronic uncertainties,  $R_c$  and  $R_n$  would deviate substantially (being larger) from data, and disallowed. This is again consistent with the analysis from  $\mathcal{A}_{K^+\pi^0} - \mathcal{A}_{K^+\pi^-}$  as well as  $b \rightarrow s\ell^+\ell^-$  and  $B_s$  mixing, and with our findings for  $S_{\pi K}$  and  $S_{\phi K}$ .



**Figure 3:** (a)  $S_{\eta K^0}$ , (b)  $\mathcal{B}(\eta K^0)$ , (c)  $S_{\omega K^0}$ , (d)  $\mathcal{B}(\omega K^0)$ , (e)  $S_{\rho K^0}$ , (f)  $\mathcal{B}(\rho K^0)$  vs  $\phi_{sb}$ , in QCDF at NLO. The curves are for the “S3” hadronic parameter settings. The light gray regions correspond to varying the hadronic parameters  $X_A$ ,  $X_H$  and  $\lambda_B$  over the range indicated in eq. (10). The dark gray regions are obtained by varying the hadronic parameters over the same range as above, but keeping only the values for which the branching ratios are within  $3\sigma$  of the experimental central values.

Both CDF and D0 Collaborations have recently updated the results on  $B_s$  mixing, giving both lower and upper bound that are consistent with SM expectations [23]. This does not affect our results on  $\Delta S_f$  in any substantial way. As pointed out in ref. [11], the combined bounds of  $b \rightarrow sl^+\ell^-$  and  $\Delta m_{B_s}$  implies the latter should be consistent with SM expectation, but predicts  $\sin 2\Phi_{B_s} < 0$  in the 4th generation scenario, since best

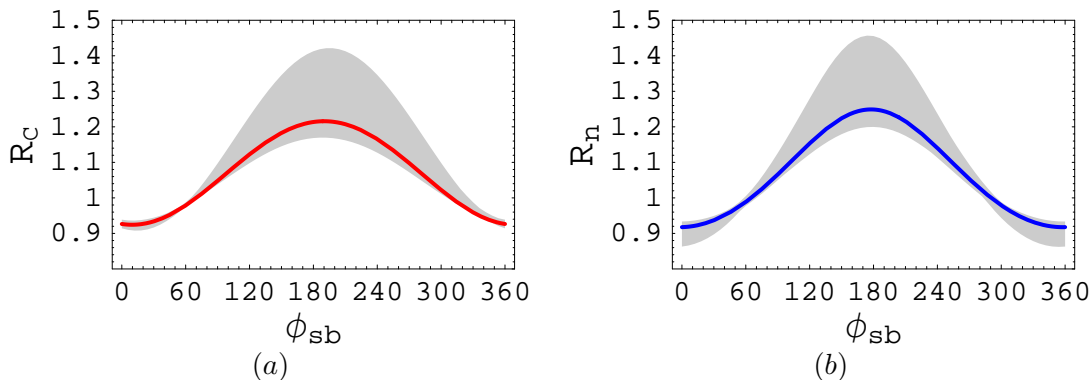




**Figure 4:** (a)  $C_{\pi K^0}$ , (b)  $C_{\eta K^0}$ , (c)  $C_{\phi K^0}$ , (d)  $C_{\omega K^0}$ , (e)  $C_{\eta' K^0}$ , (f)  $C_{\rho K^0}$  vs  $\phi_{sb}$ , in QCDF at NLO. The curves are for the “S3” hadronic parameter settings. The light gray regions correspond to varying the hadronic parameters  $X_A$ ,  $X_H$  and  $\lambda_B$  over the range indicated in eq. (10). The dark gray regions are obtained by varying the hadronic parameters over the same range as above, but keeping only the values for which the branching ratios are within  $3\sigma$  of the experimental central values.

consistency is when  $\phi_{sb}$  is nearly imaginary. With the measurement of  $B_s$  mixing, one could then hope to test the CPV prediction.

$S_f$  has been studied experimentally in quite a few other modes such as  $f = f_0(980)K_S$ ,  $\omega K^0$  [1], as well as 3-body modes such as  $f = K\bar{K}K$  and  $K_S\pi^0\pi^0$ . The interest in  $S_{\eta K}$ ,  $S_{\omega K}$  and  $S_{\rho^0 K}$  have been stressed [3, 5]. We have studied these modes as presented in figure 3



**Figure 5:** Ratio of branching ratios (a)  $R_c$  and (b)  $R_n$  for  $B^+$  and  $B^0$  decay to  $K\pi$ , as defined in ref. [8], vs  $\phi_{sb}$ , for QCDF at NLO. The curves and the light gray regions are obtained as in figure 2.

and found the effect of hadronic uncertainties to be more significant. Thus, experimental studies in these modes would shed little light on NP parameters, except that  $S_{\rho^0 K} > \sin 2\phi_1$  is likely realized. The theory for 3-body modes is even less developed. Similarly, DCPV depends sensitively on hadronic phases, and much theoretical work is currently ongoing to elucidate these. As shown in figure 4 our studies do show that DCPV in the above mentioned 2-body modes are in general consistent with data, since experimental errors are still large. The only firmly measured DCPV is in  $\mathcal{A}_{K^+\pi^-}$ , while ref. [11] has demonstrated that the 4th generation may help resolve the  $\mathcal{A}_{K^+\pi^0} - \mathcal{A}_{K^+\pi^-} \neq 0$  problem.

Finally, we note from figure 2 that for QCDF the experimental central values are unattainable once the branching ratio is constrained to within  $3\sigma$  of experiment. (Note, however, from figure 1, that our approximate PQCD result could fit the  $S_{\phi K}$  central value.) If the experimental central values for  $S_{\phi K}$ ,  $S_{\pi K}$  and  $S_{\eta' K}$  persist, more work on factorization models seem needed to shed further light on whether the 4th generation, or other New Physics, could account for the observed effect.

In summary, we have studied in this work the effect of a 4th generation model on the TCPV parameter  $S_f$  for  $f = \pi K^0$ ,  $\phi K^0$  and  $\eta' K^0$ . We have shown, using QCDF at NLO, that the NP effects on these  $S_f$ 's are rather robust against hadronic uncertainties. This robustness may be generic to a large class of NP models. We found that the same 4th generation parameters that explain  $\mathcal{A}_{K^+\pi^0} \sim 0$  while  $\mathcal{A}_{K^+\pi^-} \simeq -11\%$ , can give the correct trend in  $S_f$ . However, we also showed that  $S_{\eta' K^0}$ ,  $S_{\pi K^0}$  and to a lesser degree  $S_{\phi K^0}$  are predicted to be closer to  $\sin 2\phi_1$  than the current data indicate. Due to the robustness of the  $S_f$ , better measurements could provide an important test of the 4th generation model as well as other NP models.

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